Graph Reductions and Transformations

Lino Demasi

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Plan

1. Basic Graph Definitions
2. Important Theorems
3. Reductions
4. Transformations
5. What I'm Trying To Do

Basic Graph Definitions

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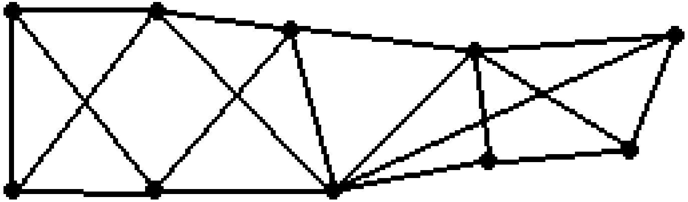

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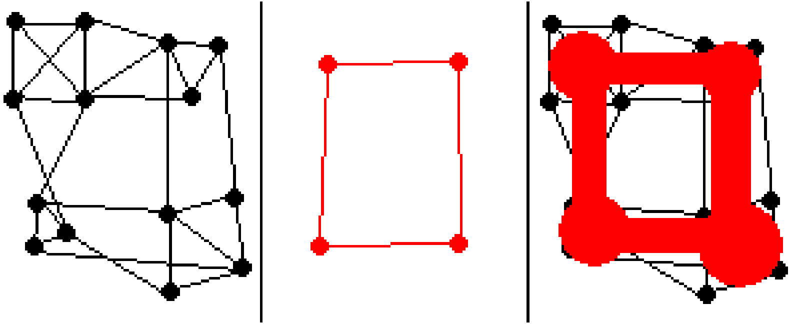

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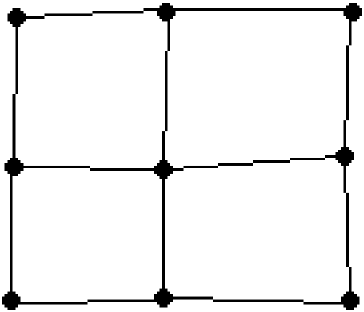

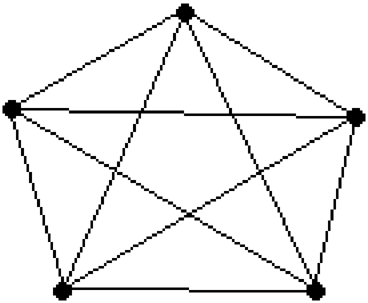

Important Theorems

Graph Minors Theorem:

- If you have a family F of graphs such that if a graph $G \in F$ then all minors of $G \in F$, then there exists a finite list of graphs L such that a graph H is in F iff H does not contain any graph of L as a minor.

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Kuratowksi's Theorem:

- A graph is planar iff it does not have K_5 or $K_{3,3}$ as a minor.

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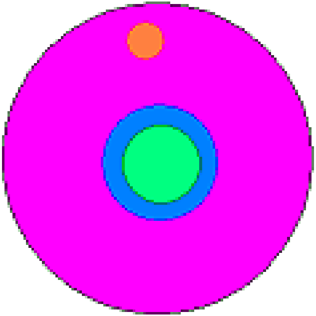

Pink = all graphs Green = our family Blue = boundary set
Orange = graphs wearing pants

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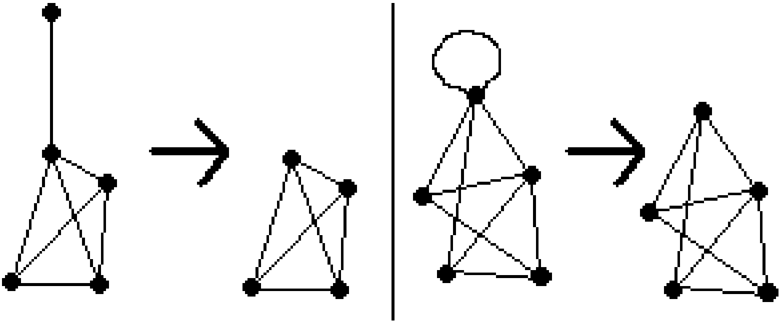

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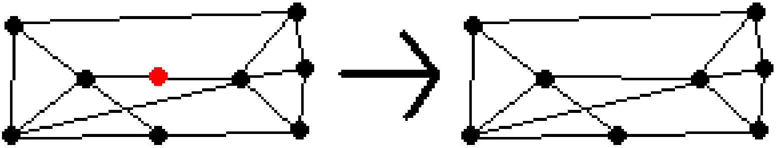

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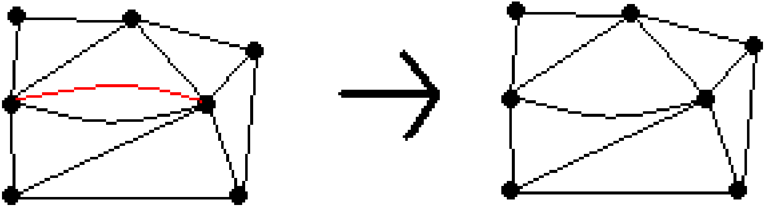

Transformations

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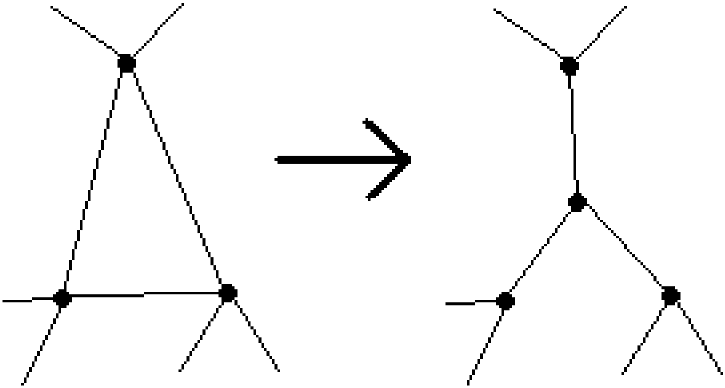

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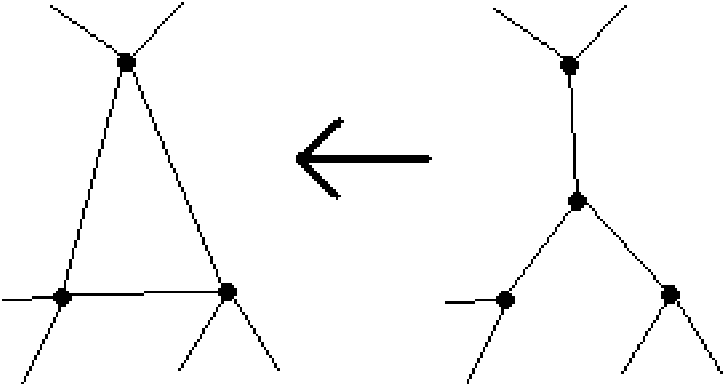

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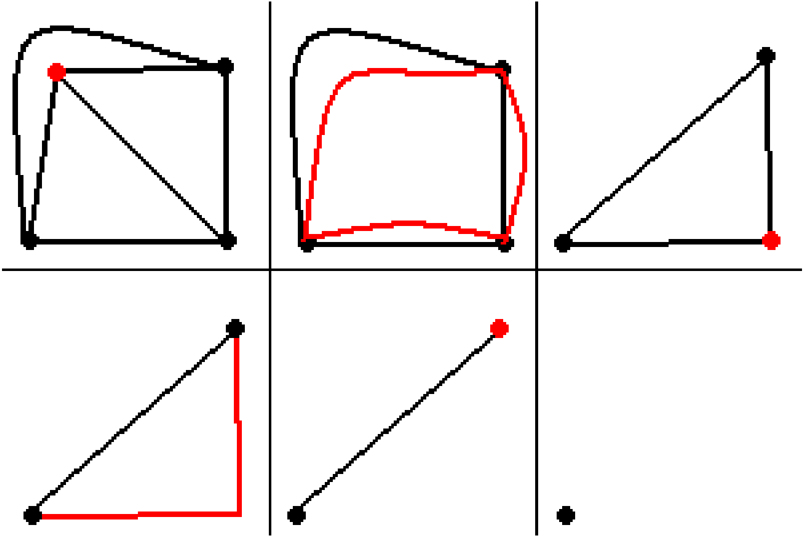

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- Yu (2004,2006) found 68 billion graphs in the boundary set. They fall into 20 families.

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- Doing this by considering how many neighbours the extra vertex has, and assuming that the planar graph is a grid.

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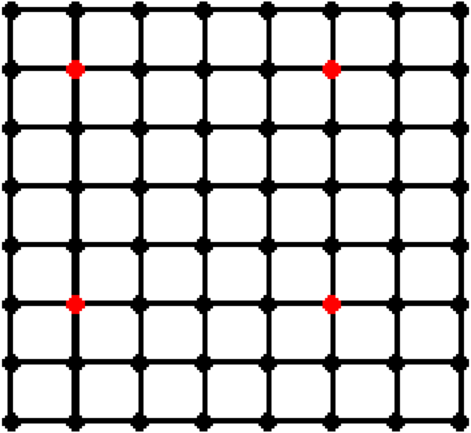

The red vertices indicate where the extra vertex is attached to.

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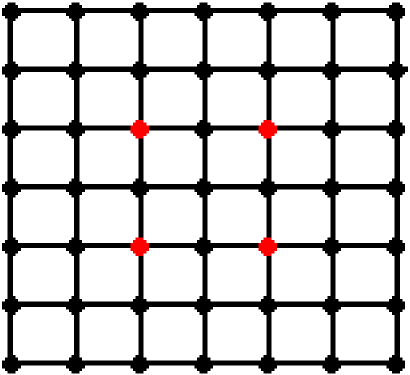

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- It is okay if I don't find more minors, and simply show that the existing list is complete.

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- The examination and classification work will take a long time.
- There is a termination condition, since if I get to far away from planar, then the graph must not be reducible.
- Determining whether I've found a graph in Yu's list is not necessarily easy, so it would be nice to find a way to check this efficiently.

End Credits

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