Research Notes - Acyclic Dichromatic Number

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Definition. For a digraph D we define the acyclic dichromatic number to be the minimum number of colours needed to colour D such that each colour class induces an acyclic subgraph.

Theorem. *Almost all torunaments T have $\chi(T) \geq \frac{n}{2\log_2 n}$*

Proof. Let $k = 2\log_2 n$.

$$\begin{aligned} P[\alpha(D) > k] &\leq \binom{2n}{k} P[\text{fixed subset is acyclic}] \\ &\leq \binom{n}{k}(2k)! \left(\frac{1}{2}\right)^{\binom{k}{2}} \\ &\leq \frac{n^k}{k!} k! \left(\frac{1}{2}\right)^{\frac{k^2-k}{2}} \\ &\leq n^{2\log_2 n} \left(\frac{1}{n}\right)^{2\log_2 n - 1} \\ &= n \end{aligned}$$

Adding a constant gives the result.

□

Theorem. *Every tournament T has $\chi(T) \leq \frac{n}{\log_2 n}(1 + o(1))$*

Theorem. *Almost every graph has $\chi(G) \leq \frac{n}{2\log_2 n}$*