Research Notes - d-regular Vertex Transitive Graphs

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For $A \subset V(G)$,

Definition. $\partial A = \{N(A) - A\}$

Definition. $\delta A = \{e = uv, u \in \partial A, v \in A\}$

Definition. $\text{diam}(A) := \max\{\text{dist}_G(u, v) | u, v \in A\}$

Definition. $\text{depth}(A) := \max\{\text{dist}_G(\partial A, v) | v \in A\}$

Lemma. *$diam(A) < |\partial A|(2depth(A) + 1)$*

Proof. Order elements of the boundary set. Path of length at most 2depth(A) in each, plus moving between. □

Definition. A block of imprimitivity is a set $B \in \Omega$ s.t. $B^g = B$ or $B^g \cap B = \phi$ for all group actions g.

Definition. The collection of sets is called a system of imprivity.

Theorem. *(Mader) If G is finite, vertex transive, $d-$regular then G is $d-$edge connected.*

Proof. Suppose for contradiction that $|\delta(A)| < d$ and choose a set A that is minimal with this property. Clearly $|A| \leq \frac{|V|}{2}$.

For any graph and for sets $A, B \subset V$, we have the submodularity property

$$|\delta(A)| + |\delta(B)| \geq |\delta(A \cup B)| + |\delta(a \cap B)|$$

If A is not a block of imprimitivity then it contradicts submodularity.

If A is such a block, then each vertex has s neighbours internal to the block and $d - s$ neighbours external to the block. $(s + 1)(d - s) \geq d$, which is a contradiction. □